SU-ITP-97-12
April 1997
hep-th/9704086

A Proposal for the Wrapped Transverse Five Brane in M(atrix) Theory

Edi Halyo*

Department of Physics
Stanford University
Stanford, CA 94305-4060

We propose a state in $5+1$ super Yang–Mills theory which corresponds to the wrapped transverse five brane of M(atrix) theory on T^5. This state is a magnetic flux quantized in units of $1/g_6^2$ through a plane defined by one side of the box and a new direction which is not manifest.

* halyo@dormouse.stanford.edu

Recently, there has been great interest in the matrix model formulation of M theory[1]. In this framework, M theory in the light–cone coordinates is described by an infinite number of zero branes and their interactions due to open strings stretched between them. This is a $0+1$ dimensional $U(N)$ super Yang–Mills (SYM) field theory with sixteen supersymmetries which is obtained by the dimensional reduction of $D = 10$ SYM theory with $\mathcal{N} = 1$ supersymmetry. By now, the matrix model has passed a number of consistency checks such as long range interactions between gravitons, membranes etc.[2], the formulation of different types of strings[3] and the presence of U dualities[4]. Fundamental states of M theory such as gravitons and membranes have been easily identified but the five branes of the theory turned out to be more elusive. The longitudinal five brane has been constucted as a background [5] but the transverse five brane seems to be absent in the matrix model[6]. (However, see [7] for work related to the M(atrix) theory five brane.)

When M theory is compactified on T^5, the transverse five brane can be wrapped completely on the five torus. In this case, one expects the transverse five brane to appear in the spectrum of the matrix model on the same footing as the other pointlike BPS states. Since matrix model on T^d is described by $U(N)$ SYM theory in a d dimensional box[1,8], the transverse five brane must be one of the BPS states of this theory. This theory has sixteen BPS states and fifteen of them have been identified in terms of the Abelian electric and magnetic fluxes of the SYM theory[9]. In this note, we propose a candidate for the BPS state of the SYM theory which corresponds to the transverse five brane wrapped on T^5. This state is simply a magnetic field which is quantized in units of $1/g_6^2$, where g_6 is the $5+1$ dimensional SYM coupling with dimension of length. The direction of the magnetic field (or flux) is given by one side of the box and a new dimension which is not manifest. This new dimension is the one which opens up in a $4+1$ dimensional SYM theory as one increases the gauge coupling g_5^2[10]. It is in some sense conjugate to the instanton (or soliton) number of the $4+1$ SYM theory. The energy of this state SYM theory matches the light–cone energy which corresponds to the wrapped transverse five brane.

In the following, we first describe the $5+1$ dimensional $U(N)$ SYM theory and its manifest fifiteen BPS states which are given in terms of the electric and magnetic fluxes. We then describe the BPS state which corresponds to the transverse five brane and calculate its mass. We end with a short discussion of our results.

As mentioned above, matrix model on T^5 (with compact dimensions of length L_1, L_2, L_3

, L_4, L_5) is described by an $U(N)$ SYM theory in a five dimensional box with sides (parametrized by $\sigma_i, \quad i = 1, \ldots, 5$) [9]

$$\Sigma_i = \frac{(2\pi)^3 \ell_{11}^3}{R L_i} \tag{1}$$

and coupling constant

$$g_6^2 = \frac{(2\pi)^9 \ell_{11}^9}{R^2 L_1 L_2 L_3 L_4 L_5} \tag{2}$$

with dimension of area. The Lagrangian of the model is given by

$$\mathcal{L} = \int^V d^5\sigma \quad Tr(-\frac{1}{4g_6^2} F_{\mu\nu}^2 + (D_\mu \phi_i)^2 + g_6^2 [\phi_i, \phi_j]^2 + fermionic \quad terms) \tag{3}$$

where $V = \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5$ is the volume of the box, $\mu = 0, \ldots, 5$, $i = 6, \ldots, 9$ and all fields are in the adjoint representation of $U(N)$. M(atrix) theory on T^5 has sixteen pointlike BPS states. Five of these are Kaluza–Klein or momentum states with mass

$$M = \frac{2\pi}{L_i} \tag{4}$$

and light–cone energy $H = M^2/2p_{11} = M^2 R/2N$

$$H = \frac{(2\pi)^2 R}{2N L_i^2} \tag{5}$$

In the SYM theory, these are described by $U(1)$ electric fluxes which satisfy

$$\epsilon_{ijklm} E_i \Sigma_j \Sigma_k \Sigma_l \Sigma_m = \frac{2\pi n_i}{N} \tag{6}$$

Using

$$H_e = \frac{N}{2} g_6^2 E^2 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \tag{7}$$

we find precisely the light–cone energy in eq. (5). In addition, there are ten BPS states which correspond to membranes wrapped on the ten two tori of T^5 with mass (the membrane

tension is $1/(2\pi)^2\ell_{11}^3$)

$$M = \frac{L_i L_j}{(2\pi)^2 \ell_{11}^3} \tag{8}$$

and light–cone energy

$$H = \frac{(L_i L_j)^2 R}{(2\pi)^4 \ell_{11}^6 2N} \tag{9}$$

These are described by the ten $U(1)$ magnetic fluxes which satisfy the quantization condition

$$B_{ij}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N} \tag{10}$$

Using

$$H_m = \frac{N}{2} g_6^{-2} B^2 \Sigma_1\Sigma_2\Sigma_3\Sigma_4\Sigma_5 \tag{11}$$

we find precisely the light–cone energy in eq. (9).

The sixteenth BPS state corresponds to the wrapped transverse five brane with mass (the five brane tension is $1/(2\pi)^5\ell_{11}^6$)

$$M = \frac{L_1 L_2 L_3 L_4 L_5}{(2\pi)^5 \ell_{11}^6} \tag{12}$$

This state could not be identified in the $5+1$ SYM theory in terms of the usual Abelian fluxes[9]. We propose that the transverse five brane is described by a magnetic flux quantized in units of $1/g_6^2$ (similar to the other magnetic fluxes quantized in units of inverse the area of the faces of the five dimensional box), i.e.

$$B g_6^2 = \frac{2\pi n}{N} \tag{13}$$

We also need to specify the direction of this magnetic flux. This can be done by realizing that

$$g_6^2 = g_5^2 \Sigma_5 = \Sigma\Sigma_5 \tag{14}$$

where $g_5^2 = (2\pi)^6\ell_{11}^6/RL_1L_2L_3L_4$ is the coupling constant of the $U(N)$ SYM in a four dimensional box. It has been argued that g_5^2 corresponds to a new dimension of this box (parametrized by σ and with size Σ) which opens up as the coupling increases[10]. The

reason for this is as follows. The $4+1$ dimensional SYM theory has instantons (or solitons) which correspond to the longitudinal five branes of M theory with energy n/g_5^2. Since $g_5^2 = \Sigma$ has dimension of length, these can be interpreted as momentum modes of a new dimension with size Σ. Thus the magnetic field in eq. (13) is in the direction $\sigma\sigma_5$ which is not manifest in the five dimansional box. Substituting this into eq. (11) we get

$$H_m = \frac{N}{2}\frac{(2\pi)^2}{g_6^2}\frac{1}{g_6^4}\Sigma_1\Sigma_2\Sigma_3\Sigma_4\Sigma_5 \tag{15a}$$

$$= \frac{R(L_1L_2L_3L_4L_5)^2}{(2\pi)^{10}\ell_{11}^{12}} \tag{15b}$$

This reproduces the correct mass for the wrapped five brane. The above description of the transverse five brane is in some sense similar to the description of wrapped transverse membranes. These are described by magnetic fluxes in eq. (10) whereas longitudinal membranes wrapped on RL_i are given by momentum modes (or photons) in the box, i.e. states with $p_i = n/\Sigma_i$. On the other hand, the longitudinal five brane is described by the momentum mode $p = n/\Sigma$. Thus, in analogy, we expect that the transverse five brane is given by the magnetic flux

$$B_{\sigma\sigma_5}\Sigma\Sigma_5 = 2\pi n \tag{16}$$

which is precisely eq. (13). A possible gauge field configuration which describes the transverse five brane is

$$A_5 = \frac{2\pi}{g_6^2 N}\sigma I_{N\times N} \tag{17}$$

Note that among all the BPS states only the longitudinal and transverse five branes are described in terms of the new direction σ.

The sixteen BPS states are in the spinor representation of the U duality group $SO(5,5;Z)$. However, in the five dimensional box only the $SL(5;Z)$ part of this is manifest as a geometrical symmetry. Under $SL(5;Z)$ the sixteen BPS states decompose as a 10 (the wrapped membranes), a 5 (the Kaluza–Klein states) and a singlet which is the wrapped five brane. The five brane is a singlet because the factor $\Sigma\Sigma_5 = \ell_{11}^9/R^2L_1L_2L_3L_4L_5$ which defines it is invariant under permutations of Σ_i (or L_i). The electric and magnetic fluxes transform among themselves and form the 5 and 10 representations. The transformations $\Sigma \leftrightarrow \Sigma_i$ exchange the five brane with one of the membranes. These are some of the generators of $SO(5,5;Z)$ which are not in $SL(5;Z)$.

In this note we proposed a state in the $5+1$ SYM theory as a candidate for the wrapped transverse five brane of M theory. This is simply a magnetic flux through a plane defined by one side of the box and a new dimension which is not manifest. The interpretation of this dimension which is roughly conjugate to instanton number of $4+1$ SYM theory is not clear. For this reason, it is not possible to write down the configuration of the five brane only in terms of the variables of the SYM theory in a five dimensional box. This may also be the reason for the absence of the five brane central charge in the matrix model. Clearly more needs to be done to show that this state in fact describes the transverse five brane. First and foremost a better understanding of the σ direction in terms of the five dimensional box variables is needed. This will enable us to write down a configuration which corresponds to the five brane. Once the configuration is known, one can try to show that membranes can end on these configurations and/or calculate the Berry's phase obtained by taking a membrane around the five brane.

Acknowledgements

We would like to thank Moshe Rozali and Lenny Susskind for discussions.

REFERENCES

1. T. Banks, W. Fischler, S. Shenker and L. Susskind, hep-th/9610043.
2. O. Aharony and M. Berkooz, hep-th/9611215, G. Lifschytz and S. Mathur, hep-th/9612087; G. Lifschytz, hep-th/ 9612123.
3. L. Motl, hep-th/9701025; T. Banks and N. Seiberg, hep-th/9702187; R. Dijkgraaf, E. Verlinde and H. Verlinde, hep-th/9703030; Y. Imamura, hep-th/9703077; T. Banks and L. Motl, hep-th/9703218; D. Lowe, hep-th/9704041.
4. S. Sethi and L. Susskind, hep-th/9702101; L. Susskind, hep-th/9611164; O.J. Ganor, S. Ramgoolam and W. Taylor, hep-th/9611202.
5. M. Berkooz and M. R. Douglas, hep-th/9610236.
6. T. Banks, N. Seiberg and S. Shenker, hep-th/9612157.
7. A. Fayazuddin and D. Smith, hep-th/9703208; G. Lifschytz, hep-th/9703201.
8. W. Taylor,hep-th/9611042.
9. W. Fischler, E. Halyo, A. Rajaraman and L. Susskind, hep-th/9703102.
10. M. Rozali, hep-th/9702136.